AP Calculus AB Summer Assignment

Welcome to AP Calculus AB! **Please read ALL of the information that follows.**

Directions for Completing the Summer Packet:
Please complete the problems to the best of your ability. Do not copy the answers to someone else's summer assignment -- this is grounds for cheating. Make sure you write **in pencil** and your writing is legible. If you need to use additional sheets of paper for completion, please do so.

Other Notes:
- This assignment is to be completed by **the first day of school**.
- You will need a TI-84 graphing calculator. You may purchase one yourself or borrow one from the math department during the first full week of school. **Please do not use any other calculator -- you will be required to switch.**
- I will be available to answer questions throughout the summer, and particularly during the final week leading up to school. More information will be provided about this later. Should you have any questions, please contact me (hpritchett@solebury.org).

I look forward to this year in AP Calculus AB!

Hannah Pritchett

AP Calculus AB Summer Packet

The following formulas and identities will help you complete this packet.

Additionally, students are expected to know ALL of these by memory for the course.

<u>Linear forms</u>: Slope-intercept: $y = mx + b$ Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$ Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope is undefined)

Parallel → Equal slopes Perpendicular → Slopes are opposite reciprocals

<u>Quadratic forms</u>: $y = ax^2 + bx + c$ $y = a(x - h)^2 + k$ $y = a(x - p)(x - q)$

<u>Reciprocal Identities</u>: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

<u>Quotient Identities</u>: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

<u>Pythagorean Identities</u>: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

<u>Double Angle Identities</u>: $\sin(2x) = 2\sin x \cos x$ $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

<u>Exponential Properties</u>: $x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$ $x^0 = 1$ for all $x \neq 0$

$\frac{x^a}{x^b} = x^{a-b}$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$ $\sqrt[b]{x^n} = x^{n/b}$ $x^{-n} = \frac{1}{x^n}$

<u>Logarithms</u>: $y = \log_a x$ is equivalent to $a^y = x$

<u>Logarithmic Properties</u>: $\log_b mn = \log_b m + \log_b n$ $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \cdot \log_b m$ If $\log_b m = \log_b n$, then $m = n$ $\log_a n = \frac{\log_b n}{\log_b a}$

AP Calculus AB Summer Packet

For #1-10, write an equation for each line in point-slope form.

1. Containing $(4, -1)$ with a slope of $\frac{1}{2}$

2. Crossing the x-axis at $x = -3$ and the y-axis at $y = 6$

3. Containing the points $(-6, -1)$ and $(3,2)$

4. Write an equation of a line passing through $(5, -3)$ with an undefined slope.

5. Write an equation of a line passing through $(-4,2)$ with a slope of 0.

6. Write an equation in point-slope form passing through $(0,5)$ with a slope of $\frac{2}{3}$.

7. Write an equation of a line passing through $(2,8)$ that is parallel to $y = \frac{5}{6}x - 1$.

8. Write an equation of a line passing through $(4,7)$ that is perpendicular to the y-axis.

9. Write an equation of a line with an x-intercept of $(2,0)$ and a y-intercept of $(0,3)$.

10. Write an equation of a line passing through $(6, -7)$ that is perpendicular to $y = -2x - 5$.

For #11-18, solve each equation for x. Note that some equations with have a specific value, but most will have a solution for x in terms of other variables. For example: $x = \frac{a+b}{c}$ would be a solution.

11. $x^2 + 3x = 8x - 6$

12. $\frac{2x-5}{x+y} = 3 - y$

13. $3xy + 6x - xz = 12$

14. $A = ax + bx$

15. $cx = vx$

16. $r = t - x(z - y)$

17. $\frac{3+x}{5-x} = 6 + y$

18. $\frac{y+2}{4-x} = 4(2 - z)$

For #19-24, solve each quadratic by factoring.

19. $x^2 - 4x - 12 = 0$

20. $x^2 - 6x + 9 = 0$

21. $x^2 - 9x + 14 = 0$

22. $x^2 - 36 = 0$

23. $9x^2 - 1 = 0$

24. $4x^2 + 4x + 1 = 0$

For #25-29, evaluate the following knowing that $f(x) = 5 - \frac{2x}{3}$ and $g(x) = \frac{1}{2}x^2 + 3x$.

25. $f\left(\frac{1}{2}\right) =$

26. $g(-2) =$

27. $f(1) + g(0) =$

28. $f(0) \cdot g(0) =$

29. $\frac{g(-6)}{f(-6)} =$

Recall function composition notation $(f \circ g)(x)$ is the same thing as $f(g(x))$.

For #30-39, use $f(x) = x^2 - 1, g(x) = 3x,$ *and* $h(x) = 5 - x$ *to find each composite function.*

30. $(f \circ g)(x) =$

31. $(g \circ f)(x) =$

32. $(f \circ f)(4) =$

33. $(g \circ h)(-4) =$

34. $(f \circ (g \circ h))(1) =$

35. $(g \circ (g \circ g))(5) =$

36. $f(g(x-1)) =$

37. $g(f(x^3)) =$

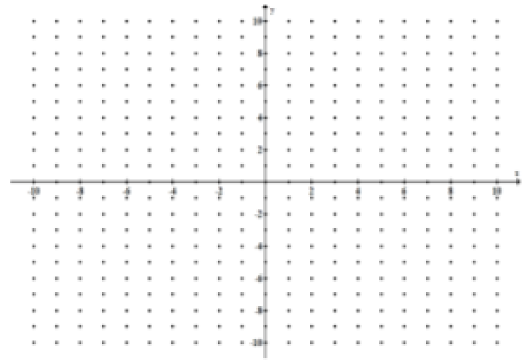
38. $\frac{f(x+h)-f(x)}{h} =$

39. The expression in the previous problem is very significant and important in Calculus. Think back to Pre-Calculus… what is the name of that expression?

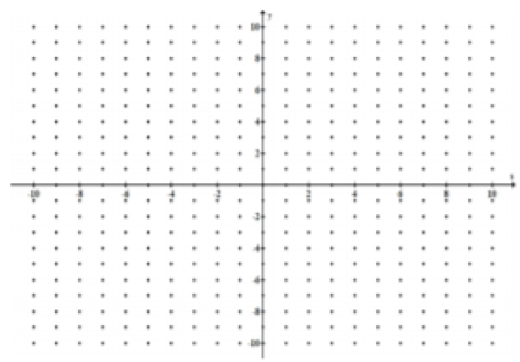
____

For #40-42, graph each piecewise function.

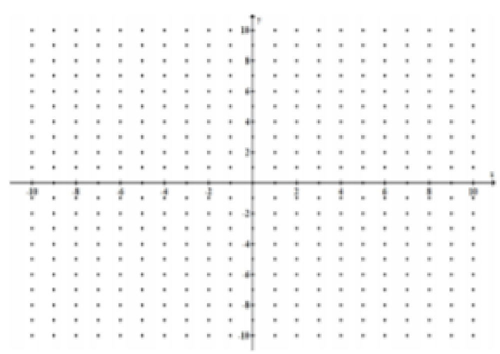
40. $f(x) = \begin{cases} x+3 & ; \ x < 0 \\ -2x+5 & ; \ x \geq 0 \end{cases}$



41. $g(x) = \begin{cases} \frac{1}{2}x & ; \ -4 \leq x \leq 2 \\ 2x-3 & ; \quad x > 2 \end{cases}$



42. $h(x) = \begin{cases} |x| & ; \ x \leq 1 \\ 2-|x-2| & ; \ x > 1 \end{cases}$



*An **exponential equation** is an equation in which the variable is in the exponent. To solve an exponential equation, you must use a logarithm to solve it.*

For #43-47, solve each exponential equation, rounding answers to the nearest thousandth. Note that some equations can be solved by writing each side as the same base instead of using a logarithm.

43. $5^x = \frac{1}{5}$

44. $6^x = 1296$

45. $6^{2x-7} = 216$

46. $5^{3x-1} = 49$

47. $10^{x+5} = 125$

For #48-51, simplify each expression without the use of a calculator.

48. $e^{\ln 4} =$

49. $e^{2\ln 3} =$

50. $\ln e^{9} =$

51. $5\ln e^{3} =$

For #52-57, solve each equation using natural logarithm. Round answers to the nearest thousandth.

52. $e^{x} = 34$

53. $3e^{x} = 120$

54. $e^{x} - 8 = 51$

55. $\ln x = 2.5$

56. $\ln(3x - 2) = 2.8$

57. $\ln(e^{x}) = 5$

For #58-66, find each exact value of the expression using the Unit Circle. ***NO CALCULATOR!***

58. $\sin 120° =$ ________

59. $\cos \frac{11\pi}{6} =$ ________

60. $\tan 225° =$ ________

61. $\sin\left(-\frac{2\pi}{3}\right) =$ ________

62. $\sin 150° =$ ________

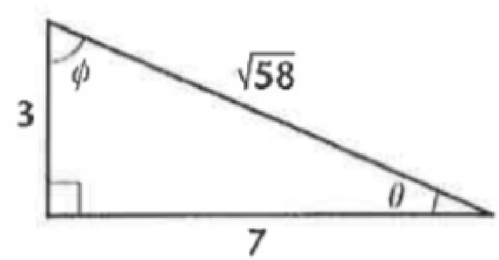
63. $\tan \frac{7\pi}{4} =$ ________

64. $\csc\left(\frac{\pi}{4}\right) =$ ________

65. $\sec(-210°) =$ ______

66. $\cot\left(\frac{5\pi}{4}\right) =$ ________

For #67-74, evaluate each trigonometric expression using the right triangle provided.



67. $\sin\theta =$ ________

68. $\cos\theta =$ ________

69. $\tan\phi =$ ________

70. $\csc\phi =$ ________

71. $\sec\theta =$ ________

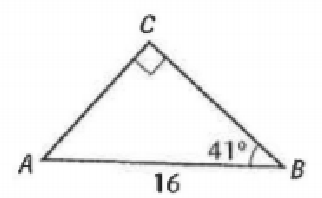
72. $\cot\theta =$ ________

73. $\sin\phi =$ ________

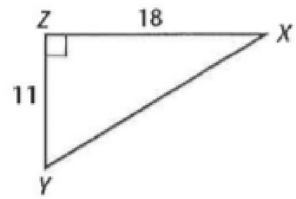
74. $\sec\phi =$ ________

For #75-76, solve each triangle, rounding all angles and sides to the nearest thousandth. Recall that "solve a triangle" means to find all missing sides and angles.

75.



76.



For #77-84, evaluate each inverse trigonometric function. ***NO CALCULATOR!***

77. $\sin^{-1}\left(\frac{1}{2}\right) =$ _______

78. $\sin^{-1}(-1) =$ _______

79. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ _______

80. $\tan^{-1}(\sqrt{3}) =$ _______

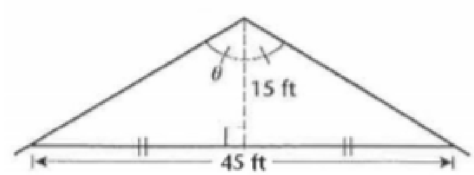
81. $\tan^{-1}(-1) =$ _______

82. $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right) =$ _______

83. $\sin\left(\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right) =$ ______

84. $\sin^{-1}(\cos(0)) =$ _______

85. Find the angle at the peak of the roof, as shown in the picture. Round to the nearest thousandth.



86. Explain how the graph of $f(x)$ and $f^{-1}(x)$ compare.

Recall that to find an inverse of a function, simply switch the x and y and solve for y. We use the notation $f^{-1}(x)$ to define the inverse of $f(x)$.

For #87-89, find the inverse of each function.

87. $g(x) = \frac{5}{x-2}$

88. $f(x) = \frac{x^2}{3}$

89. $y = \sqrt{4-x} + 1$

90. If the graph of $f(x)$ has the point $(2,7)$, then what is one point on the graph of $f^{-1}(x)$?

For #91-94, convert the inequalities in to ***interval notation****. For example, $x > 3$ becomes $(3, \infty)$.*

91. $1 < x \leq 10$

92. $x < 0$ or $x \geq 4$

93. $x \geq -2$

94. $x \geq 4$ and $x > 10$

For #95-100, find the domain and range of each function. Write the answer in interval notation. Confirm your answer by graphing the function in your graphing calculator.

95. $f(x) = \sqrt{x+5}$

D: ____________________ R: ____________________

96. $f(x) = x^2 - 5$

D: ____________________ R: ____________________

97. $f(x) = \frac{1}{x+7}$

D: ____________________ R: ____________________

98. $f(x) = \frac{5}{x^2+1}$

D: ____________________ R: ____________________

99. $f(x) = \sqrt{x^2+5}$

D: ____________________ R: ____________________

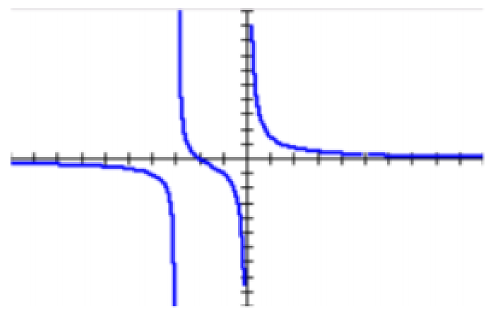
100. $g(x) = x^3 + 2x - 7$

D: ____________________ R: ____________________

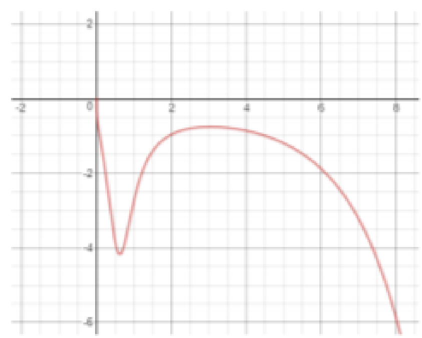
For #101-103, answer the question by referring to the function and its graph.

101. State the domain and range of $f(x) = \frac{2x^2-6x-20}{x^3-2x^2-15x}$

Hint: There is a hole.



102. Consider the function $f(x) = \frac{e^x}{\log x - x^3}$.

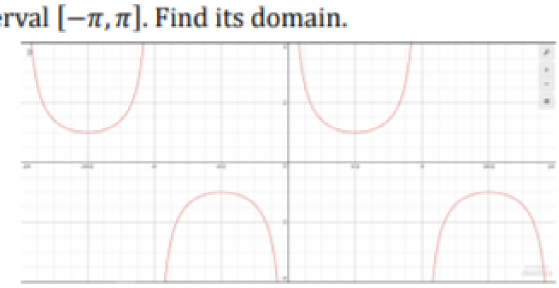


Find the maximum and minimum y-value of the function.

State the domain of $f(x)$.

State when the function is increasing and decreasing (write in interval notation).

103. Consider the function $f(x) = \csc x$ on the interval $[-\pi, \pi]$. Find its domain.



The difference quotient is defined to be $\frac{f(x+h)-f(x)}{h}$ and is a core concept for the development of calculus. For #104-107, find the difference quotient of each function.

104. $f(x) = 9x + 3$

105. $f(x) = 5 - 2x$

106. $f(x) = x^2 - 3x$

107. $f(x) = \frac{2}{x+1}$